1 **(a)** **(i)** Factorise $8x^3+27$. [2]

(ii) Hence determine, showing all necessary working, the number of real roots of the equation $8x^3+27=0$. [3]

(b) The coefficient of x^3 of a cubic polynomial, f(x), is 4 and that the roots of the equation f(x) = 0 are -1, 3 and k. Given that f(x) has a remainder of 60 when divided by $x-2$, find the value of k. [3]

2 A curve has the equation $y=3x^2+4x+c$, where c is a constant.

(i) In the case where $c=-15$, find the range of values of x for $y>0$. [2]

(ii) Find the range of values of c such that the curve lies completely above the $x-$axis. [2]

(iii) Find the value of c for which the line $2y+x=1$ is a tangent to the curve. [4]

3 **(a)** Without using a calculator, find the exact value of $\cot\left(\frac{2\pi}{3}\right)$. [2]

(b) Given that $\sin A=-\frac{5}{13}$ and $\tan A>0$, find without using a calculator, the numerical value of

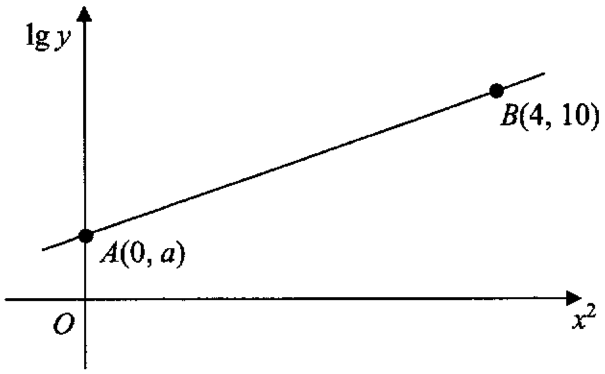
(i) $\sec A$, [2]

(ii) $\tan(-A)$. [1]

4 **(i)** Sketch the graphs of $y=2x^{\frac{1}{3}}$ and $y^2=2x$ on the same diagram for $x\geq 0$. [2]

(ii) The two graphs $y=2x^{\frac{1}{3}}$ and $y^2=2x$ intersect at (0, 0) and a point A. Find the coordinates of A. [3]

5 The variables x and y are related in such a way that, when $\lg y$ is plotted against x^2, a straight line passing through the point $A(0, a)$ and the point $B(4, 10)$ is obtained, as shown in the diagram.



Given that the line has a gradient of 2, find

(i) the value of a, [1]

(ii) the expression for y in terms of x, [1]

(iii) the values of x when $y = 1000$. [2]

6 The equation of a circle C is $x^2 + 6x + y^2 - 10y = 66$.

(i) Find the radius and the coordinates of the centre of the circle. [3]

(ii) Given that PQ is the diameter of the circle, where P is the point (5, 11), find the coordinates of the point Q. [2]

(iii) Find the equation of the circle C_1, which is a reflection of the circle C in the line $x = -1$. [2]

7 **(i)** Given that $u = 3^x$, express $3^{2x-1} = 3^x + 6$ as an equation in u. [1]

(ii) Hence find the value(s) of x for which $3^{2x-1} = 3^x + 6$. [2]

(iii) Explain why the equation $3^{2x-1} = 3^x - k$ has no solution if $k > 0.75$. [3]

8 **(a)** Solve $\log_4(x-2)-\log_4(x+2)=1+\log_4\frac{1}{9}$. [4]

(b) Solve the equation $\lg x=\log_x 1000$, giving your answer to 2 significant figures. [4]

9 **(a)** One root of the equation $8x^2-bx+1=0$ is twice the other root.
Find the possible value(s) of b. [4]

(b) Given that the roots of $2x^2-6x+3=0$ are α and β, find the quadratic equation whose roots are $\frac{3}{\alpha^2}$ and $\frac{3}{\beta^2}$. [5]

END OF PAPER

1 A liquid is allowed to cool after being heated. The temperature, θ °C of the liquid, t seconds after being removed from the heat is given by $\theta = 25 + 80e^{-0.03t}$.

(i) Find the initial value of θ. [1]

(ii) Find the time taken for the liquid to cool to 60 °C. [2]

(iii) Explain why θ does not fall below 25 °C. [1]

2 Express $\dfrac{7x+4}{(x^2+5)(x-2)}$ in partial fractions. [5]

3 (i) The line $y - 2x + 9 = 0$ intersects the curve $x^2 + y^2 + xy + 3x = 46$ at the points R and Q. Find the coordinates of points R and Q. [4]

(ii) Find the equation of the perpendicular bisector of RQ. [3]

4 A right-angled triangle has base $(4 + 2\sqrt{3})$ cm and area $(6\sqrt{3} - 2)$ cm^2. Find the height of the triangle, giving your answer in the form $(a\sqrt{3} + b)$ cm where a and b are integers. [4]

5 (a) Find all the angles between 0° and 360° inclusive which satisfy the equation

(i) $\tan(2x + 60°) = 1.2$, [3]

(ii) $\tan y = 2 \sin y$. [4]

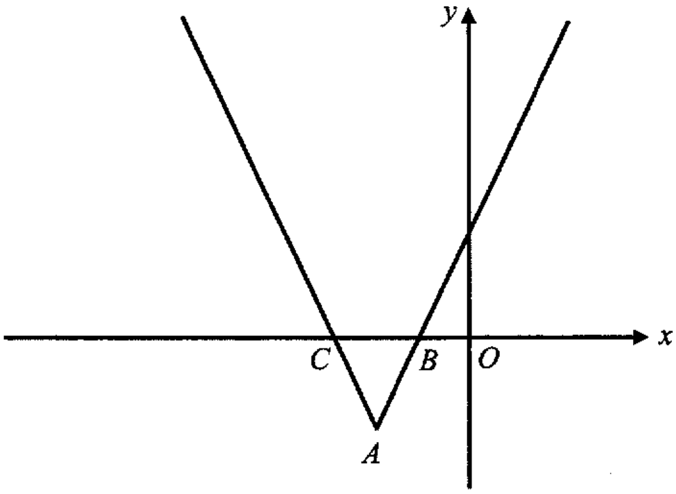
(b) Given that $0 \le x \le 2\pi$, find all the angles which satisfy the equation $2\cos^2 x = 1$. [3]

6 **(a)** Solve the simultaneous equations

$$25^x \div 5^{y+1} = 1,$$
$$\log_6 x = 1 - \log_6 y.$$ [6]

(b) Solve $3^x = e^{2x-5}$. [3]

7 The diagram shows part of the graph of $y = |3x+5| - 2$.



(i) Find the coordinates of the points A, B and C. [3]

(ii) Solve the equation $|3x+5| - 2 = x + 4$. [3]

(iii) State the number of solution(s) of the equation $|3x+5| - 2 = -3$. [1]

8 **(a)** Find the coefficient of x^2 in the binomial expansion of $\left(x - \frac{1}{3x}\right)^8$. [4]

(b) Find the first 3 terms in the expansion, in ascending powers of x, of

(i) $(1+3x)^7$,

(ii) $(2-x)^4$.

Hence, find the coefficient of x^2 in the expansion of $(1+3x)^7(2-x)^4$. [6]

9 The function f is defined by $f(x) = 3\cos 2x + 1$ for $0° \le x \le 180°$.

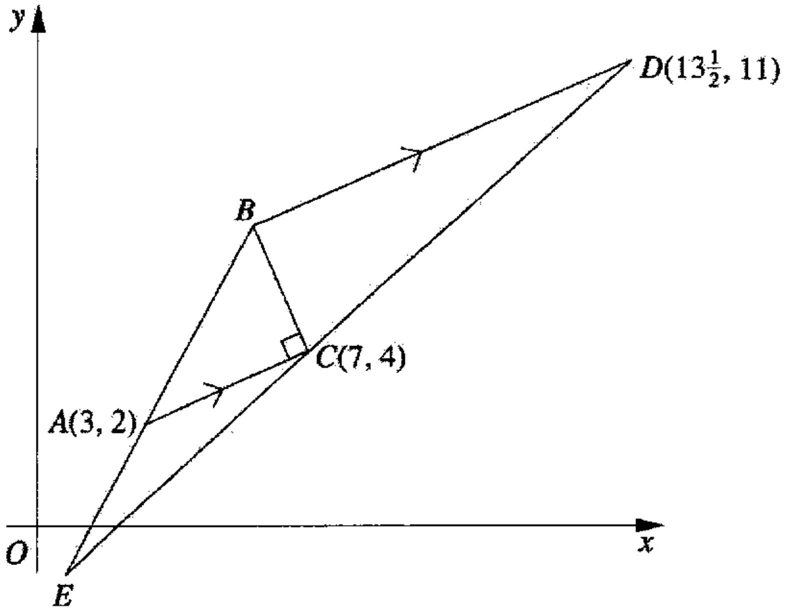
(i) State the amplitude of f. [1]

(ii) State the period of f. [1]

(iii) Find the x-coordinates of the points where the curve meets the x-axis. [3]

(iv) Sketch the graph of $y = 3\cos 2x + 1$ for $0° \le x \le 180°$. [2]

10 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a triangle ABC in which the coordinates of the points A and C are (3, 2) and (7, 4) respectively. $\angle ACB = 90°$. The line BD is parallel to AC and D is the point $\left(13\frac{1}{2}, 11\right)$. The lines BA and DC are extended to meet at E.

Find

(i) the equation of line BD, [2]

(ii) the coordinates of B, [4]

(iii) the ratio of the area of the quadrilateral $ABDC$ to the area of the triangle BCD. [3]

11 The table below shows experimental values of two variables, x and y, which are connected by an equation of the form $ay = x + \frac{b}{x}$, where a and b are constants.

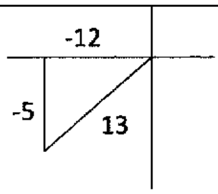
x	2	4	6	8
y	0.6	0.95	1.3	1.7

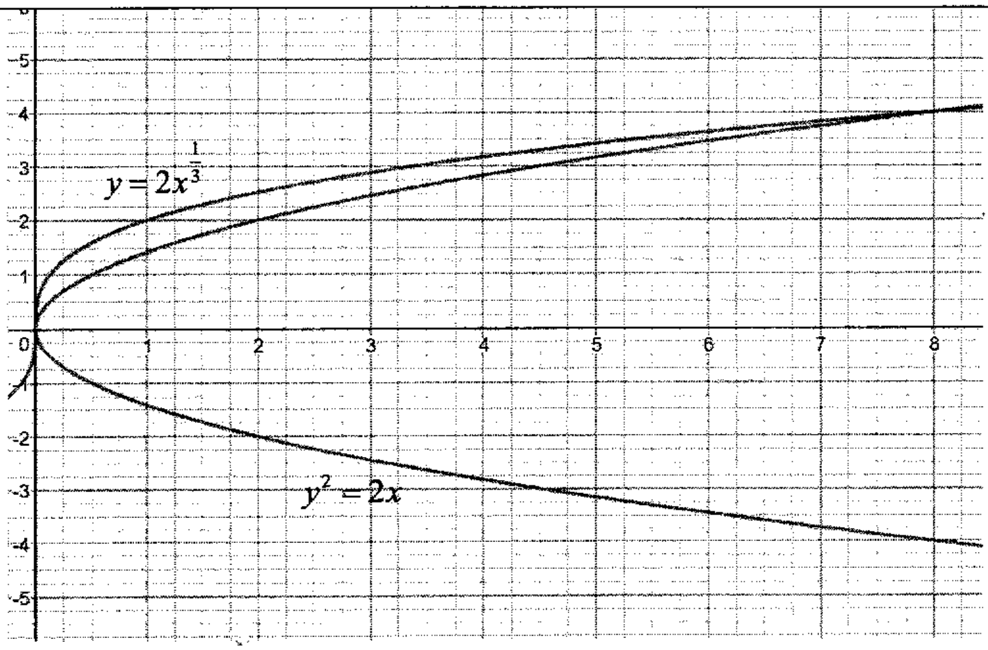
(i) Plot xy against x^2 and draw a straight line graph. Use your graph to estimate the value of each of the constants a and b. [6]

(ii) Using your graph, find the value of y when $x = 7$. [2]

END OF PAPER

Qn	Solutions	
1ai [2]	$8x^3+27=(2x)^3+3^3$ $=(2x+3)\left[(2x)^2-(2x)(3)+3^2\right]$ $=(2x+3)(4x^2-6x+9)$	M1 A1
aii [3]	$8x^3+27=0$ $(2x+3)(4x^2-6x+9)=0$ $x=-1.5$ or $4x^2-6x+9=0$ $D=(-6)^2-4(4)(9)$ $=-108<0$ hence no real roots $\therefore$ No of real roots = 1 (i.e $x=-1.5$)	 M1 M1 A1
(b) [3]	$f(x)=4(x+1)(x-3)(x-k)$ $f(2)=60$ $4(2+1)(2-3)(2-k)=60$ $-12(2-k)=60$ $2-k=-5$ $k=7$	M1 M1 A1
2(i) [2]	$y=3x^2+4x-15>0$ $(3x-5)(x+3)>0$ $x<-3$ or $x>\frac{5}{3}$	 M1 A1
(ii) [2]	Curve lies above x-axis i.e. $b^2-4ac<0$ $(4)^2-4(3)c<0$ $16-12c<0$ $c>1\frac{1}{3}$	 M1 (D<0) A1
(iii) [4]	$2y+x=1\Rightarrow y=\frac{1-x}{2}$ Subt into $y=3x^2+4x+c$ $\frac{1-x}{2}=3x^2+4x+c$ $1-x=6x^2+8x+2c$ $6x^2+9x+2c-1=0$ line is a tangent to curve, D = 0 $9^2-4(6)(2c-1)=0$ $81-48c+24=0$ $-48c=-105$ $c=\frac{35}{16}=2\frac{3}{16}$	 M1 (eliminates y) M1 (correct quad) M1 (any use of D=0) A1

3(a) [2]	$\cot\left(\frac{2\pi}{3}\right)=\frac{1}{\tan\left(\frac{2\pi}{3}\right)}$	M1
	$=-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$	A1
b(i) [2]	$\sec A=\frac{1}{\cos A}$ $=\frac{1}{-\frac{12}{13}}$ 	M1
	$=-\frac{13}{12}$ or $-1\frac{1}{12}$	A1
(ii) [1]	$\tan(-A)=-\tan A=-\frac{5}{12}$	B1

4(i) [2]	 B1 – Graph of $y=2x^{\frac{1}{3}}$ B1 – Graph of $y^2=2x$	

(ii) [3]	$\left(2x^{\frac{1}{3}}\right)^2 = 2x$	M1
	$4x^{\frac{2}{3}} = 2x$	
	$x^{\frac{2}{3}} = \frac{1}{2}x$	
	$x^2 = \left(\frac{1}{2}x\right)^3$	
	$x^2 - \frac{1}{8}x^3 = 0$	M1
	$x^2\left(1 - \frac{1}{8}x\right) = 0$	
	$x = 0$ (NA) or 8	
	$y = 4$	
	Coordinates of $A = (8,\ 4)$	A1
5(i) [1]	$A(0,a), B(4,10)$ $grad = \frac{10-a}{4-0} = 2$ $10 - a = 8$ $a = 2$	B1
(ii) [1]	$\lg y = 2x^2 + 2$ $y = 10^{2x^2+2}$	B1
(iii) [2]	$\lg 1000 = 2x^2 + 2$ $3 = 2x^2 + 2$ $x^2 = \frac{1}{2}$ $x = \pm 0.707$	B2
6(i) [3]	$x^2 + 6x + y^2 - 10y = 66$ Centre = (-3, 5), $radius\ = \sqrt{9 + 25 - (-66)}$ $= 10 units$	B1 M1 A1
(ii) [2]	Midpoint of PQ = centre of circle $\left(\frac{5+a}{2}, \frac{11+b}{2}\right) = (-3,5)$ $\therefore \frac{5+a}{2} = -3 \Rightarrow a = -11$ $\frac{11+b}{2} = 5 \Rightarrow b = -1$ $Q(-11,-1)$	M1 A1
(iii) [2]	New centre = (1, 5), $r = 10$ $(x-1)^2 + (y-5)^2 = 100$.	M1 A1

7(i) [1]	$3^{2x-1}=3^x+6$ $\frac{(3^x)^2}{3}=3^x+6$ $\frac{u^2}{3}=u+6$ or $u^2-3u-18=0$	B1(any appropriate eqn in u)
(ii) [2]	$u^2-3u-18=0$ $(u-6)(u+3)=0$ $u=6$ or $u=-3$(Rej) $3^x=6$ $x\lg 3=\lg 6$ $x=1.63$ (3sf)	M1 A1
(iii) [3]	$3^{2x-1}=3^x-k$ $\frac{u^2}{3}=u-k$ $u^2-3u+3k=0$ No solutions $\Rightarrow D<0$ $(-3)^2-4(1)(3k)<0$ $9-12k<0$ $-12k<9$ $k>0.75$	M1 M1 M1
8(a) [4]	$\log_4(x-2)-\log_4(x+2)=1+\log_4\frac{1}{9}$ $\log_4\frac{x-2}{x+2}=\log_4 4+\log_4\frac{1}{9}$ $\log_4\frac{x-2}{x+2}=\log_4\frac{4}{9}$ $\therefore\frac{x-2}{x+2}=\frac{4}{9}$ $9x-18=4x+8$ $5x=26$ $x=5.2$	M1(division law) M1(get rid of log) M1 A1
(b) [4]	$\lg x=\log_x 1000$ $\lg x=\frac{\lg 10^3}{\lg x}$ $(\lg x)^2=3$ $\lg x=\sqrt{3}$ or $-\sqrt{3}$ $x=10^{\sqrt{3}}$ or $10^{-\sqrt{3}}$ $x=54$ or 0.019 (2sf)	M1 M1 A2(2 sig fig)

9(a) [4]	$8x^2 - bx + 1 = 0$ Let roots be α and 2α. sum of roots: $\alpha + 2\alpha = -\frac{-b}{8}$ $3\alpha = \frac{b}{8}$ $b = 24\alpha$ product of roots: $\alpha(2\alpha) = \frac{1}{8}$ $\alpha^2 = \frac{1}{16}$ $\alpha = \pm\frac{1}{4}$ $\therefore b = 24\alpha = \pm 6$	M1 M1 M1 A1
(b) [5]	$2x^2 - 6x + 3 = 0$ $\alpha + \beta = 3 \qquad \alpha\beta = \frac{3}{2} = 1.5$ Sum of roots $= \frac{3}{\alpha^2} + \frac{3}{\beta^2}$ $= \frac{3(\alpha^2 + \beta^2)}{(\alpha\beta)^2}$ $= \frac{3\left[(\alpha+\beta)^2 - 2\alpha\beta\right]}{(\alpha\beta)^2}$ $= \frac{3\left[(3)^2 - 2(1.5)\right]}{(1.5)^2}$ $= 8$ Product of roots $= \left(\frac{3}{\beta^2}\right)\left(\frac{3}{\alpha^2}\right)$ $= \frac{9}{(1.5)^2} = 4$ $\therefore$ Equation is $x^2 - 8x + 4 = 0$	M1 M1-sum of roots formula A1 M1 A1

AMKSS Final Exam A.Math Paper 2

Answer Scheme

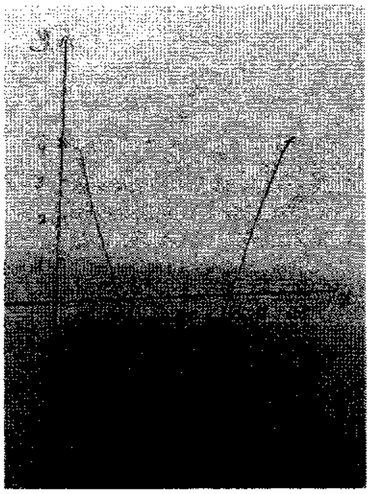
Qn	Answer	Mark Allocation
1(i)	When $t=0$, $\theta = 25+80e^{0}$ $\theta = 105^{o}C$	B1
1(ii)	$60 = 25+80e^{-0.03t}$ $e^{-0.03t} = \frac{35}{80}$ $-0.03t = \ln\left(\frac{35}{80}\right)$ $t \approx 27.6s$	M1 A1
1(iii)	Since $e^{-0.03t} > 0$ $80e^{-0.03t} > 0$ $25+80e^{-0.03t} > 25$ θ does not fall below $25°C$	B1
2.	$\frac{7x+4}{(x^2+5)(x-2)} = \frac{Ax+B}{x^2+5}+\frac{C}{x-2}$ $7x+4 = (Ax+B)(x-2)+C(x^2+5)$ When $x=2; 18=9C$ $C=2$ When $x=0$; $4=-2B+2(5)$ $B=3$ When $x=1$; $11=(A+3)(-1)+2(6)$ $A=-2$ $\frac{7x+4}{(x^2+5)(x-2)} = \frac{-2x+3}{x^2+5}+\frac{2}{x-2}$	M1 A1 A1 A1 A1

3(i)	$x^2+y^2+xy+3x=46-(1)$ $y-2x+9=0-(2)$ Subst (1) into (2) $x^2+(2x-9)^2+x(2x-9)+3x=46$ $x^2+4x^2-36x+81+2x^2-9x+3x-46=0$ $7x^2-42x+35=0$ $x^2-6x+5=0$ $(x-5)(x-1)=0$ $x=5$ or $x=1$ $y=1$ or $y=-7$ $R(5,1)$ and $Q(1,-7)$	M1 M1 M1 A1
3(ii)	$M_{RQ}=2$ M of perpendicular bisector $=-\frac{1}{2}$ Midpoint $(3,-3)$ $y=-\frac{1}{2}x+c$ At$(3,-3)$ $-3=-\frac{1}{2}x+c$ $c=-\frac{3}{2}$ $y=-\frac{1}{2}x-\frac{3}{2}$	M1 M1 A1
4	$\frac{1}{2}\times(4+2\sqrt{3})\times h=6\sqrt{3}-2$ $h=\frac{6\sqrt{3}-2}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$ $h=\frac{12\sqrt{3}-6\times3-4+2\sqrt{3}}{4-3}$ $h=14\sqrt{3}-22$	M1 M1 M1 A1
5(a)(i)	$\tan(2x+60°)=1.2$ Acute$\angle=50.19443$ $2x+60°=50.19443(NA),230.19443,$ $410.19443,590.19443,770.19443$ $x=85.1°,175.1°,265.1°,355.1°$	M1 M1 A1

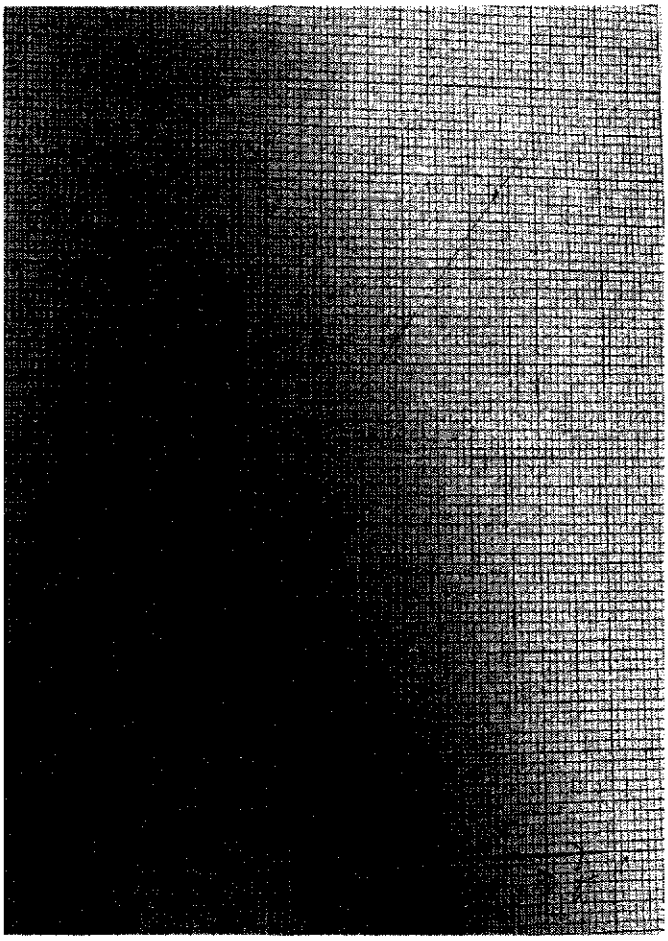
5(a)(ii)	$\frac{\sin y}{\cos y} = 2\sin y$ $\sin y - 2\sin y\cos y = 0$ $\sin y(1-2\cos y) = 0$ $\sin y = 0$ or $\cos y = \frac{1}{2}$ Acute$\angle = 0$ Acute$\angle = 60°$ $y = 0°, 180°, 360°$ $y = 60°, 300°$	M1 M1 M1 A1
5(b)	$2\cos^2 x = 1$ $\cos^2 x = \frac{1}{2}$ $\cos x = \pm\sqrt{\frac{1}{2}}$ Acute$\angle = 0.785398$ $x = 0.785, 2.36, 3.93, 5.50$	 M1 M1 A1
6(a)	$25^x \div 5^{y+1} = 1$ $5^{2x} \div 5^{y+1} = 5^0$ $2x - y - 1 = 0$ $2x - y = 1 - (1)$ $\log_6 x = 1 - \log_6 y$ $\log_6 x + \log_6 y = 1$ $\log_6(xy) = 1$ $xy = 6$ $y = \frac{6}{x} - (2)$ Subst (2) into (1) $2x - \frac{6}{x} = 1$ $2x^2 - x - 6 = 0$ $(2x+3)(x-2) = 0$ $x = -1\frac{1}{2}$ or $x = 2$ $y = -4$(NA) $y = 3$	 M1 M1 M1 M1 M1 A1

6(b)	$3^x = e^{2x-5}$ $\ln 3^x = \ln e^{2x-5}$ $x\ln 3 = 2x-5$ $x(\ln 3-2) = -5$ $x = \frac{-5}{\ln 3 - 2}$ $x = 5.55$	M1 M1 A1
7(i)	$\lvert 3x+5\rvert - 2 = 0$ $\lvert 3x+5\rvert = 2$ $3x+5=2$ or $3x+5=-2$ $x=-1$ $\quad$ $x=-\frac{7}{3}$ $B(-1,0)$, $C\left(-2\frac{1}{3},0\right)$ $3x+5=0$ $x=-\frac{5}{3}$ $A\left(-1\frac{2}{3},-2\right)$	B1, B1 B1
7(ii)	$\lvert 3x+5\rvert - 2 = x+4$ $\lvert 3x+5\rvert = x+6$ $3x+5=x+6$ or $3x+5=-x-6$ $x=\frac{1}{2}$ $\quad$ $x=-2\frac{3}{4}$	M1 M1 A1
7(iii)	0	B1

8(a)	General Term $= {}^8C_r(x)^{8-r}\left(-\frac{1}{3x}\right)^r$ $= {}^8C_r x^{8-r}(x)^{-r}\left(-\frac{1}{3}\right)^r$ $= {}^8C_r x^{8-2r}\left(-\frac{1}{3}\right)^r$ $8-2r=0$ $r=3$ ${}^8C_3 x^{8-2(3)}\left(-\frac{1}{3}\right)^3$ $=-\frac{56}{27}x^2$ Coefficient of $x^2 = -\frac{56}{27}$	M1 M1 M1 A1
8(b)(i)	$(1+3x)^7$ $= {}^7C_0(1)^7(3x)^0 + {}^7C_1(1)^6(3x)^1 + {}^7C_2(1)^5(3x)^2$ $=1+21x+189x^2$	M1 A1
8(b)(ii)	$(2-x)^4$ $= {}^4C_0(2)^4(-x)^0 + {}^4C_1(2)^3(-x)^1 + {}^4C_2(2)^2(-x)^2$ $=16-32x+24x^2$ $(1+21x+189x^2)(16-32x+24x^2)$ $=21x(-32x)+16(189x^2)+24x^2$ $=-672x^2+3024x^2+24x^2$ $=2376x^2$ Coefficient of $x^2 = 2376$	M1 A1 M1 A1
9(i)	3	B1
9(ii)	180°	B1

9(iii)	$y=0$ $3\cos 2x+1=0$ $\cos 2x=-\frac{1}{3}$ $\text{Acute}\angle=70.52877937$ $2x=109.5°,250.5°$ $x=54.7°,125.3°$	 M1 M1 A1
9(iv)	 Correct shape, turning point Correct x, y intercepts	 B1 B1
10(i)	$M_{AC}=\frac{1}{2}$ $M_{BD}=\frac{1}{2}$ Equation of BD: $y=\frac{1}{2}x+c$ At $\left(13\frac{1}{2},11\right)$ $11=\frac{1}{2}\left(13\frac{1}{2}\right)+c$ $c=\frac{17}{4}$ $y=\frac{1}{2}x+\frac{17}{4}$	 M1 A1

10(ii)	$M_{BC}=-2$ Equation of BC: $y=2x+c$ At $(7,4)$ $4=-2(7)+c$ $c=18$ $y=-2x+18$ $-2x+18=\frac{1}{2}x+\frac{17}{4}$ $x=5\frac{1}{2}$ $y=7$ $B\left(5\frac{1}{2},7\right)$	M1 M1 M1 A1
10(iii)	Area of $ABDC$ $=\frac{1}{2}\begin{vmatrix}3 & 7 & 13\frac{1}{2} & 5\frac{1}{2} & 3\\ 2 & 4 & 11 & 7 & 2\end{vmatrix}$ $=22.5$units Area of BCD $=\frac{1}{2}\begin{vmatrix}5\frac{1}{2} & 7 & 13\frac{1}{2} & 5\frac{1}{2}\\ 7 & 4 & 11 & 7\end{vmatrix}$ $=15$units Ratio $=3:2$	 M1 M1 A1

11(i)	$ay = x + \frac{b}{a}$	
	$xy = \frac{x^2}{a} + \frac{b}{a}$	
	Plot xy against x^2	M1
	Straight line	M1
	Gradient $= \frac{1}{a}$	
	Intercept $= \frac{b}{a}$	
	$\frac{b}{a} = 0.4$	M1
	$b = 1.94 \pm 0.2$	A1
	$\frac{1}{a} = \frac{14-0.4}{66-0}$	M1
	$\frac{1}{a} = 0.206060606$	
	$a = 4.85 \pm 0.2$	A1
		

11(ii)	$x^2 = 49$ $xy = 10.45$ $y = \frac{10.45}{7}$ $y = 1.49 \pm 0.2$	M1 A1